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NAVWEPS REPORT 7636 NOTS TP 2648 COPY 93

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HORIZONTAL AIR-MASS TABLES

By

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ABSTRACT. The illumination of artificial earth satellites during their motion into the earth's shadow is a problem of current interest. For heights above the surface greater than 20 km, the tangential solar ray incident on a satellite in a non-absorbed region of the spectrum is attenuated principally by Rayleigh scattering. The attenuation is a function of the total number of molecules, or, if the molecular weight remains constant, of the air mass along the path of the ray that is tangential to any level, h, above the surface.

Tables are given in terms of mass of air and number of molecules for a vertical path originating at h, a horizontal path tangent to h, the horizontal quantities relative to a unit atmosphere, and the rate of change with respect to h. The tables cover an altitude range of 0 to 100 km, calculated for an atmosphere terminated at 200 km. These tables are intended primarily for use above a nominal altitude of 20 km, where refraction, which was not allowed for in the computations, has a negligible effect.

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U.S. NAVAL ORDNANCE TEST STATION

China Lake, California

15 March 1961

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AN ACTIVITY OF THE BUREAU OF NAVAL WEAPONS

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Technical Director

FOREWORD

This work is part of the program for determining the ozone distribution in the atmosphere by photometric observations of artificial satellites--particularly Echo I (1960 Iota one)--and was supported by Local Project 507 and Bureau of Naval Weapons Task Assignment 360FR106-2161-R01101001. It has been reviewed for technical accuracy by E. V. Ashburn, O. N. Strand, and J. C. Smith.

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NOTS Technical Publication 2648 NAVWEPS REPORT 7636

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INTRODUCTION

The orbiting of large balloon reflectors such as Echo I has provided a new tool for investigating the composition of the upper atmosphere. In addition to its primary utility as a radio communications reflector, the satellite is also particularly useful as an extra-atmospheric_reflector of light having known reflecting properties. Although Echo I was launched in an orbit which was continuously illuminated by the sun, the orbit has a regression rate, relative to the shadow axis of the earth, of approximately four degrees per day. Examination of the geometry involved shows that the orbital plane intersects the shadow cone of the earth about 60 percent of the time. On these 'shadow passes' the solar ray illuminating the satellite scans the atmosphere just before the satellite enters the shadow and just after it leaves the shadow on each rotation. At any time that the illuminating ray is traversing the atmosphere, the illuminance at any wavelength is a function of the distribution of any constituents that absorb at that particular wavelength, and is also a function of the molecular density distribution, which alters the intensity by Rayleigh scattering. Since a horizontal path through any atmospheric layer is much longer than a vertical path, it is possible to measure the distribution of minor atmospheric constituents.

Two methods of analysis are available. An integral technique (Ref. 1) has been used for lunar eclipse measurements of the ozone distribution, and has been modified for application to satellite measurements (Refs. 2 and 3). Secondly, a matrix method, which can more readily be programmed for computers, has also been devised (Ref. 4).

Although the methods can be applied to the determination of the atmospheric density distribution, it is of immediate interest to calculate the Rayleigh attenuation in order to separate the absorption and scattering effects. If no absorption occurs the illuminance at the satellite is given by

$$I'(\lambda) = I_O(\lambda)e^{-k_R(\lambda)M}$$

where $I_0(\lambda)$ is the extraterrestrial solar intensity at wavelength λ , $k_R(\lambda)$ is the Rayleigh scattering cross section or coefficient, and M is the total number of molecules or the air mass along the ray from the sun to the satellite. An older tabulation of M-vs-altitude (Ref. 5) proved unsatisfactory because only the lower levels of the atmosphere were considered. Another tabulation (Ref. 6) gives air mass data to 150 km with rocket density data obtained at White Sands up to 1952. Since more recent density data are available, we felt that a new calculation of M would be useful.

It was decided to neglect the effect of refraction in the tables included in this report because in the region of interest (20 to 100 km) the errors are small. However, for completeness the data have been presented from the surface to 100 km. The user should thus exercise some caution in use of the data at the lowest levels. We hope to present the data corrected for refraction at a later date.

CALCULATIONS

We can define 'air mass' as the total mass-per-unit area along a specified path through the earth's atmosphere. The particular air mass calculated here we have called 'horizontal air mass'. The path in this case is a line through the atmosphere which passes the earth (assumed spherical) at some minimum distance, h_{Ω} , as shown in Fig. 1. .

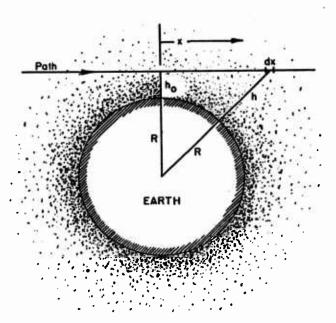


FIG. 1. Horizontal Air-Mass Geometry.

The air mass M(h_o) is then given by

$$M(h_o) = \int_{-\infty}^{\infty} \rho(h) dx$$
 (1)

where $\rho(h)$ is the mass density of the air at height h. We have assumed that the atmosphere is radially symmetric. Equation (1) can then be written as

$$M(h_0) = 2 \int_0^\infty \rho(h) dx$$
 (2)

because of this symmetry.

We now wish to introduce has the variable of integration in place of x. From the geometry of Fig. 1 we have

$$x^2 + (R+h_0)^2 = (R+h)^2$$
 (3)

and then

$$dx = \frac{(R+h)dh}{\sqrt{(R+h)^2 - (R+h_0)^2}} = \frac{(R+h)dh}{\sqrt{(h-h_0)(2R+h+h_0)}}$$
(4)

Equation (2) now reads

$$M(h_o) = 2 \int_{h_o}^{\infty} \frac{\rho(h) (R+h)dh}{[(h-h_o)(2R+h+h_o)]^{\frac{1}{2}}}$$
 (5)

Although Eq. (5) is a convergent integral for any reasonable function $\rho(h)$, it presents difficulties if the integration is performed numerically as is done here. The upper limit presents no problem since the earth's atmosphere can be considered to terminate, for practical calculations, at some finite h.

We can dispose of the discontinuity at $h = h_0$ by the following substitution:

$$y^2 = h - h_0 \tag{6}$$

Then

$$dh = 2ydy$$

and Eq. (5) becomes

$$M(h_o) = 4 \int_0^\infty \frac{\rho(y^2 + h_o)[R + h_o + y^2] dy}{[2(R + h_o) + y^2]^{\frac{1}{2}}}$$
(7)

For practical calculations the range of y can be taken as

$$y_{\text{max}} \ge y \ge 0 \qquad \bullet \qquad (8)$$

where

$$y_{\text{max}} = [h_{\text{max}} - h_o]^{\frac{1}{2}}$$
 (9)

We shall say more about h later.

Although the density of the atmosphere is approximately an exponential function of height, this was not considered a sufficiently accurate approximation for the present purpose. Instead, the ARDC 1959 model atmosphere (Ref. 6) was used. The properties of the atmosphere are tabulated functions of height. With this density function it is necessary to integrate Eq. (7) by numerical methods.

The integration was performed with Simpson's rule. The density data of the ARDC atmosphere is tabulated at various intervals, which are larger at higher altitudes. This fact, and the fact that Simpson's rule requires equal intervals, necessitated interpolation for the values of $\rho(y^2+h_0)$. The equal intervals must of course be in the variable of integration y. Second order interpolation was used. The numerical work was done on the IBM 709.

The upper limit for h was set at either 200 km or where the density became 10^{-6} of the initial density corresponding to $h_{_{\rm O}}$, whichever occurred first. The atmospheric density was taken to be zero above 200 km. The range of $h_{_{\rm O}}$ was

$$0 \le h_0 \le 100 \text{ km} \tag{10}$$

and the air mass was computed for every 1,000 m in this interval.

Another quantity of interest in these calculations is the 'zenith air mass'. This is the air mass computed along the vertical from h to 200 km. Thus

$$M_z(h_0) = \int_{h_0}^{200,000} \rho(h) dh$$
 (11)

Again the integration was performed by Simpson's rule. In this integration no interpolation was used. The interval of integration was the interval of tabulation of h. The subroutine which performed the integration took care of the end-point automatically when there was an even number of density points at some interval of h. The basic Simpson's rule requires an odd number of points.

The 'relative air mass' was also computed. We define the relative air mass by

$$R(h_o) = \frac{M(h_o)}{M_z(0)}$$
 (12)

that is, the zenith air mass at h = 0 is used as the reference.

The final quantity which was computed was the derivative of the relative air mass with respect to height h_0 . This was computed using five-point formulas (Ref. 7).

The numerical work was checked by several means. First, the zenith air mass for h should be equal to the atmospheric pressure at that h with suitable unit conversion. A small table of air pressure and acceleration of gravity from the ARDC atmosphere, and the zenith air mass, calculated pressure, and percentage difference between the pressures are presented in Table 1. The error of about 2 in 1,000 does not seem unreasonable, and may be ascribed to errors in Simpson's rule integration and other numerical processes. The large error at h = 100 km is probably due to the termination of the atmosphere at 200 km.

As a check on the integration for the horizontal air mass, an exponential density was assumed which passed through the ARDC density at h = 0 and h = 100 km. If we set

$$\rho(h) = Ae^{-ah}, \qquad (13)$$

h (meters)	P (mb)	g (m/sec ²)	$M_{z}(h_{o})$ (gm/cm^{2})	Pcalc (mb)	Pressure difference
0	1.01325 x 10 ³	9.8067	1.0356 x 10 ³	1.0155 × 10 ³	+0.23%
20,000	5.5293 x 10 ¹	9.7452	5.6851 x 10 ¹	5.540 x 10 ⁺¹	+0.20%
50,000	8.7858 x 10 ⁻¹	9.6542	9.1223 x 10 ⁻¹	8.807 x 10 ⁻¹	+0.24%
70,000	6.0209 x 10 ⁻²	9.5942	6.2864 x 10 ⁻²	6.031 x 10 ⁻²	+0.17%
90,000	1.353 × 10 ⁻³	9.535	1.4200 x 10 ⁻³	1.354 x 10 ⁻³	+0.07%
.00,000	2.138 x 10 ⁻⁴	9.505	2.238 x 10 ⁻¹	2.127 x 10 ⁻⁴	-0.51%

Zenith Air-Mass Check TABLE 1.

Equation (5) becomes

$$M(h_o) = 2 \int_{h_o}^{\infty} \frac{Ae^{-ah} (R+h)dh}{[(h-h_o)(2R+h+h_o)]^{\frac{1}{2}}}$$
 (14)

where A = 1.2250 kg/m³ and a = 1.50 x 10^{-4} /m. Now if we neglect h with respect to R,

$$M(h_o) \cong \sqrt{2R} \int_{h_o}^{\infty} \frac{e^{-ah} dh}{(h-h_o)^{\frac{1}{2}}}$$
 (15)

In a strict sense this approximation may not seem reasonable since we are integrating h from h to . However, 'infinity' here is on the order of 200 km, as compared to the radius of the earth R at some 6,000 km. exponential function decays faster than the h2 term increases. In any event, we are after an estimate to checke the order of magnitude of the numerical work. Set

$$x^2 = h - h_0 \tag{16}$$

Then Eq. (15) reads

$$M(h_o) = 2\sqrt{2R} A e^{-ah_o} \int_0^\infty e^{-ax^2} dx = \sqrt{\frac{2\pi R}{A}} A e^{-ah_o} \qquad (17)$$

Using the same density function, the zenith air mass is

$$M_z(h_o) = A \int_{h_o}^{\infty} e^{-ah} dh = \frac{A}{a} e^{-ah} o$$
 (18)

and the unit air mass, $M_2(0)$, is

$$M_{z}(0) = \frac{A}{a} \tag{19}$$

The relative air mass is therefore

$$\frac{M(h_0)}{M_z(0)} = \sqrt{2\pi Ra} e^{-ah_0}$$
 (20)

A few numerical values are given in Table 2.

TABLE 2. Estimates of Relative Air Mass

h (meters)	Relative air mass Exponential density	Calculated
0	77.5	70.3
20,000	3.86	4.34
50,000	4.28 x 10 ⁻²	6.13 x 10 ⁻²
70,000	2.13 x 10 ⁻³	4.95 x 10 ⁻³
100,000	2.37 × 10 ⁻⁵	1.74 × 10 ⁻⁵

Exact agreement is not expected, and this calculation was performed to check the order of magnitude.

A third check was made, using an atmosphere of constant density. The integration with a finite thickness can easily be performed since the density is constant. Here

 $M(h_0) = 2 \int_0^{x_1} \rho dx = 2 x_1 \rho$ (21)

From the geometry in Fig. 2,

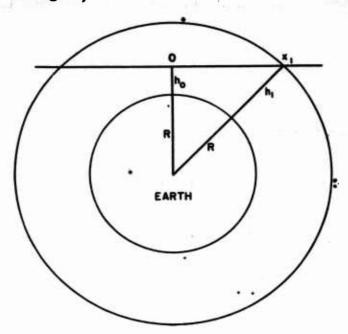


FIG. 2. Geometry for Check Calculation With Constant Density.

it is easily seen that

$$x_1^2 = (R+h_1)^2 - (R+h_0)^2 = (h_1-h_0)(2R+h_1+h_0)$$
 (22)

In this case

$$M(h_o) = 2\rho[(h_1 - h_o)(2R + h_1 + h_o)]^{\frac{1}{2}}$$
 (23)

The unit zenith air mass $M_z(0)$ is

$$M_{z}(0) = \int_{0}^{h_{o}} \rho dh = \rho h_{1}$$
 (24)

$$\frac{M(h_o)}{M_z(0)} = \frac{2}{h_1} \left[(h_1 - h_o) (2R + h_1 + h_o) \right]^{\frac{1}{2}}$$
 (25)

Table 3 shows values of the relative air mass computed by the machine program and by hand from Eq. (25).

TABLE 3. Relative Air Mass for Constant Density

h	Relative	air mass	%_difference
(meters)	Machine	Eq. 25	.pulllerence
0 .	16.077	16.084	0.044
20,000	15.261	15.271	0.066
50,000	13.945	13.956	0.079
70,000	12.982	13.003	0.162
100,000	11.409	11.417	. 0.145

The last check was made on the derivative of the relative horizontal air mass. This was done with the same formulas that were used in the machine. Agreement was noted to the last figure in the machine tabulation of the derivative for several values of h.

Strictly speaking, the air mass, M, can be used for calculating Rayleigh scattering only if the molecular weight is constant. If the latter is not constant the number density is the proper density to use. Calculations of the total number of molecules per square centimeter (integrated number density) for horizontal and Zenith paths were also made, using Equations (7), (11), and (12), and substituting number density for mass density.

The horizontal air mass and associated quantities are presented in Table 4. The two-digit number following the entries indicates the power of ten by which the number should be multiplied. Thus, .12250-02 = .12250x10⁻². The air-mass entries may be considered accurate to about 0.2% on the average if a single figure may be quoted for this quantity. The derivative error is estimated to be less than 2%.

The integrated number density and associated quantities are given in Table 5. The relative horizontal air mass from Table 4 and the relative integrated number density from Table 5 depart by (at most) 0.3% at the highest altitudes. For most practical work either table may be used.

TABLE 4. Horizontal Air Mass Vs Height

•					
17453-02	.11126+02	.11522+05	.14542+03	.22785-03	14000.
20422-02	.13016+02	.13479+05	.17009+03	.26659-03	13000.
24053-02	.15227+02	.15769+05	.19896+03	.31194-03	12000.
27205-02	.17814+02	.18448+05	.23274+03	.36480-03	11000.
29657-02	.20651+02	.21386+05	.27161+03	. 41351-03	10000.
32624-02	.23758+02	.24604+05	.31560+03	.46706-03	.0006
36068-02	.27191+02	.28159+05	.36520+03	.52578-03	.0000
39825-02	.30983+02	.32086+05	.42094+03	.59002-03	.0007
43901-02	.35166+02	.36419+05	.48340+03	. 66011-03	.0009
48300-02	.39774+02	.41190+05	.55317+03	. 73643-03	5000.
53028-02	.44837+02	.46434+05	.20+06029~	.81935-03	4000
58106-02	.50391+02	.52185+05	.71727+03 .	. 90926-03	3000.
63538-02	.56470+02	.58481+05	.81300+03	.10066-02	2000.
69336-02	.63111+02	.65358+05	.91885+03	.11117-02	1000.
75521-02	.70351+02	.72856+05	.10356+04	.12250-02	0
DERIVATIVE*	AIR MASS	CGM/SQ-CM)	CGM/SQ-CM)	CMO-UD/MDO	CMETERSO
HORIZONTAL	RELATIVE	HORIZONTAL	ZENITH	DENSITY	нетсит

* Derivative of the relative horizontal air mass per meter

TABLE 4. (contd)

· HORIZONTAL AIR MASS US. HEIGHT

HORIZONTGL AIR MASS DERIVATIVE	14917-02	12751-02	10901-02	93200-03	79687-03	68153-03	58305-03	49895-03	42733-03	· 36713-03	31488-03	27019-03	22686-03	19101-03	18130-03	
RELATIVE HORIZONTAL AIR MASS	.95105+01	.81299+01	.69496+01	.59406+01	.50779+01	.43403+01	.37092+01	.31694+01	.27071+01	.23109+01	19697+01	.16757+01	.14280+01	.12197+81	.10440+01	
HORIZONTAL AIR MASS CGM/SQ-CM)	.98492+04	.84194+04	.71971+04	.61522+04	. 52588+04	.44948+04	.38413+04	.32822+04	.28035+04	.23932+04	.20399+04	.17354+04	.14789+04	.12631+04	.10812+04	
ZENITH AIR MASS CGM/SQ-CM)	.12434+03	.10631+03	.90906+02	. 77735+02	.66476+02	.56851+02	:48622+02	.41586+02	.35569+02	.30425+02	.26026+02	.22289+02	.19122+02	.16444+02	.14166+02	
DENSITY (GM/CU-CM)	.19475-03	.16647-03	.14230-03	.12165-03	.10399-03	\$8909-04	.76015-04	.64995-04	.55575-04	.47522-04	.40639-04	.34359-04	.29077-04	.24663-04	, 20966-04	
HEIGHT (METERS)	15000.	16000.	.0007	18000.	19000.	20000.	21000.	22000.	23000.	24000.	25000.	26000.	27000.	28000.	29000.	

TABLE 4. (contd)

HORIZONTAL BIR MASS US: HEIGHT

.13044-04 .91682401 .11180-04 .79569+01 .96020-05 .69226+01 .82620-05 .60285+01 .71221-05 .52636+01 .61506-05 .45985+01 .46112-05 .40287+01 .46112-05 .35303+01 .40028-05 .35303+01 .30810-05 .24046+01 .26438-05 .24046+01

TABLE 4. (contd)

HORIZONTAL AIR MASS US. HEIGHT

	• lai		v.		ю.	4-7	la"s	40	10	10	L)	l/D	10	10	. 10	10
HORIZONTAL	DERIVATIVE	-, 14037-04	12189-04	-, 10518-04	91548-05	-,81163-05	71972-05	63725-05	56317-05	49170-05	43313-05	39218-05	35575-05	32186-05	29071-05	26214-05
RELATIVE	HORIZONTAL RIR MASS	.11185+00	.98760-01	. 87406-01	.77626-01	.68997-01	.61347-01	. 54570-01	.48575-01	. 43292-01	.38701-01	.34578-01	30838-01	.27453-01	.24392-01	.21630-01
HORIZONTÁL	RIR MRSS (GM/SQ-CM)	.11583+03	.10228+03	.90519+02	.80391+02	.71454+02	.63532+02	.56513+02	.50305+02	.44833+02	.40079+62	.35809+02	.31937+02	.28430+02	.25261+02	.22400+02
ZENITH	AIR MASS . (GM/SQ+CM)	.16552+01	.14687+01	.13001+01	.11567+01	.10245+01	.91223+00	.81014+00	.71950+00	.63902+00	.56742+00	.50298+60	.44501+00	.39294+00	.34626+00	.30448+00
DENSITY	CMD-UD/MBD	.20206-05	.17704-05	.15535-05	.13739-05	.12197-05	.10829-05	.96140-06	.85360-06	.75791-06	.67867-06	.61108-06	. 54931-06	.49293-06	.44156-06	.39482-06
HEIGHT	CMETERSO	45000.	46000.	47000.	48000.	49000.	50000.	51000.	52000.	53000.	54000.	55000.	56000.	57000.	58000.	.00008

TABLE 4. (contd)

HORIZONTAL AIR MASS US. HEIGHT

-. 23595-05 -. 17018-05 -. 94710-06 -. 56763-06 -. 49603-06 DERIVATIVE -. 21200-05 -. 19013-05 -. 15201-05 -. 13551-05 -. 12053-05 -. 10697-05 -. 83656-06 -. 64771-06 HORIZONTAL -. 73710-06 -. 43223-06 AIR MASS RELATIVE HORIZONTAL RIR MASS .76332-02 42574-02 .26558-02 .87695-02 .66258-02 .57350-02 .49490-02 36505-02 .31194-02 .16903-01 .13094-C1 11485-01 10049-01 .19141-01 14894-01 .13561+02 COM/SQ-CW> .19823+02 .17505+02 .15425+02 .11894+02 .10406+02 , 32304+01 HORIZONTAL .90818+01 79051+01 .68618+01 . 59392+01 .51253+01 .44091+01 .37805+01 .27504+01 AIR MASS .26715+00 .15467+00 .13407+00 .11590+00 CGM / SQ-CM) .23387+00 .20426+00 .17797+00 .85880-01 .73593-01 .45412-01 .38393-01 .32337-01 .99914-01 .62864-01 53522-01 ZENITH . 75424-07 .35235-06 .24745-06 .65176-07 .31384-06 .27897-06 21901-06 19341-06 .17039-06 .14975-06 .13128-06 .11478-06 10008-06 .87011-07 .56137-07 CGM/CU-CM) DENSITY CMETERSO 60000. 65000. 660000. 67000. 71000. 61000. 64000. 69000 70000. 72000. 63000. 68000. 62000. 73000. 74000. HEIGHT

TABLE 4. (contd)

		OH .	HORIZONTAL BIR I	MASS US. HEIGHT		
	HEIGHT .	DENSITY	ZENITH .	HORIZONTAL DIE MOSS	• RELATIVE	HORIZONTAL
\circ	CMETERSO	CGM/CU-CM)	COMSON-CMS	CGM/SQ-CM)		DERIVATIVE
	75000.	.48187-07	.27129-01	.23327+01	.22525-02	-,37556-06
	76000.	.41220-07	.22667-01	.19703+01	.19026-02	32533-06
	77000.	.35120-07	.18857-01	.16569+01	.15999-02	28095-06
	76000.	.29810-07	.15617-01	.13865+01	.13388-02	24201-06
	79000.	.25200-07	.12872-01	.11537+01	.11140-02	20890-06
	80000.	.21200-07	. 10557-01	.95220+00	.91946-03	18111-06
	81000.	.17340-07	.86356-02	.77862+00	.75184-03	15207-06
	82000.	.14180-07	.70649-02	.63688+00	.61498-03	12356-06
	83000.	.11600-07	57799-02	.52091+00	.50300-03	10109-06
	84000.	.94890-08	.47291-02	.42608+00	.41143-03	82682-07
	85000.	.77620-08	.38695-02	.34848+00	.33650-03	67661-07
	86000.	.63500-08	.31663-02	.28499+00	.27519-03	55371-07
	87000.	.51950-08	.25909-02	.23302+00	.22501-03	45316-07
	88000.	.42510-08	.21202-02	. 19048+00	.18393-03	37110-07
	.00068	.34780-08	.17351-02	.15563+00	.15028-03	30421-07

THELE 4. (contd)

HORIZONTAL AIR MASS US. HEIGHT

HORIZONTAL AIR MASS DERIVATIVE	24991-07	20736-07	16938-07	13456-07	10754-07	86383-08	69734-08	56556-08	46071-08	37715-08	30982-08
RELATIVE HORIZONTAL	.12267-03	. 99925-04	. 81019-04	.65919-04	.53871-64	.44217-04	.36443-04	.30153-04	.25041-04	.20867-04	. 17444-04
HORIZONTAL RIR MASS CGM/SQ-CM).	.12704+00	.10348+00	.83904-01	.68267-01	.55790-01	.45791-01	.37741-01	.31227-01	. 25933-01	.21610-01	.18065-01
ZENITH . AIR MASS	.14206-02	.11621-02	.95224-03	.78365-03	.64768-03	.03755+03	.44790-03	.37462-03	31444-03	.26484-03	.22380-03
DENSITY CGM/CU+CM)	.28460-08	.23290-08	.18770-08	.15100-08	.12210-08	. 99130-09	.80870+09	.66260-09	.54520-09	.45040-09	.37340-09
HEIGHT (METERS)	90000	91000.	92000.	93000.	94000.	95000.	96000.	97000.	98000.	900006	100000.

TABLE 5. Integrated Number Density Vs Height

17454-02	.11126+02	.23962+27 ber density per meter.	.30243+25	0. 47385+19 *Derivative of relative	14000.
20422-02	.13016+02	.28032+27	.35374+25	. 55441+19	13000.
24052-02	.15227+02	. 32794+27	.41377+25	64870+19	12000.
27205-02	.17814+02	.38366+27	.48402+25	.75864+19	11000.
29657-02	.20651+02	.44475+27	.56486+25	85993+19	10000.
32625-02	.23758+02	.51167+27	.65634+25	. 97130+19	9000.
36067-02	.27191+02	.58560+27	.75948+25	10934+20	.0000
39825-02	.30983+02	.66726+27	.87540+25	12270+20	7000.
43903-02	.35166+02	.75737+27	.10053+26	.13728+20	.0000
48299-02	.39774+02	.85660+27	.11504+26	.15315+20	<u> </u>
53027-02	.44837+02	. 96565+27	.,13120+26	.17039+20	4000.
58104-02	.50391+02	.10853+28	.14917+26	.18509+20	900
63534-02	.56470+02	.12162+28	.16907+26	.20933+20	2000.
69336-02	.63110+02	.13592+28	.19109+26	.23118+20	- - - - - - - -
75542-02	.70351+02	.15151+20	,21537+26	.25476+20.	iċ
HORIZONTAL INT NO DEN DERIVATIVE*	RELATIVE HORIZONTAL INT NO DEN	HORIZONTAL INT NO DEN CNO/SQ-CMO	ZENITH INT NO DEN . CNO/S0-CM)	NUMBER DENSITY (NO/CU-CM)	HEIGHT (METERS)

TABLE 5. (contd)

INTEGRATED NUMBER DENSITY US. HEIGHT

16131-03	.10440+01	.22484+26	.29464+24	• .43601+18	29000.
-, 19102-03	.12197+01	.26268+26	.34198+24	.51290+18	28000.
22676-03	.14280+01	.30755+26	.39772+24	.60469+18	27000.
-,27050-03	.16756+01	.36087+26	.46351+24	.71453+18	26000.
31713-03	.19702+01	.42432+26	44400440	84813418	25000.
36694-03	.23:12+01	.49776,26	.63283+24	98828+18	24000.
42729-03	.27074+01	. 58308+26	73981+24	. 11557+19	23000.
49893-03	.31696+01	.68262+26	86492+24	. 13516+19	22000.
58299-03	.37094+01	.79889+26	.10112+25	.15808+19	21000.
68152-03	.43404+01	.93478+26	.11824+25	.18496+19	20000.
79690-03	:50781+01	.10937+27	.13826+25	.21627+19	19000.
93197-03	.59408+01	.12795+27	.16167+25	.25298+19	18000.
10901-02	.69497+01	.14967+27	.18906+25	. 29593+19	17000.
12752-02	.81299+01	.17509+27	.22110+25	. 34619+19	16006.
14918-02	.95;06+01	.20483+27	 	40501+10	
INT NO DEN DERIVATIVE	HORIZONTAL INT NO DEN	INT NO DEN CHO/SQ-CMO	CNO-SQ-CNO	CNO/CU-CNO	CMETERSO
HORIZONTAL	RELATIVE	HORIZONTAL	ZENITH .		

TABLE 5. (contd)

INTEGRATED NUMBER DENSITY US. HEIGHT

HORIZONTAL INT NO DEN DERIVATIVE	12652-03	11579-03	98420-04	83826-04	71537-04	61173-04	52410-04	44981-04	38679-04	-, 33317-04	28743-04	24839-04	21497-04	18629-04	16164-04
RELATIVE HORIZONTAL INT "NO DEN	.89544+00	.76959+00	.66273+00	.57182+00	.49431+00	.42810+00	.37142+00	.32283+00	.28108+00	.24516+00	.21419+00	.18745+00	.16432+00	.14429+00	.12693+00
HORIZONTAL INT NO DEN CNO/SQ-CM)	.19285+26	.16574+26	. 14273+26	1,2315+26	.10646+26	.92198+25	. 79993+25	.69527+25	.60536+25	.52798+25	.46129+25	.40370+25	.35389+25	.31076+25	.27336+25
ZENITH INT NO DEN CNO/SQ-CM) .	.25436+24	.22002+24	.19066+24	.16553+24	.14396+24	.12543+24	.10946+24	.95688+23	.83782+23	.73474+23	.64533+23	.56766+23	.50007+23	.44117+23	.38974+23
NUMBER DENSITY CNO/CU/CM)	.37144+18	.31710+18	.27126+18	.23251+18	. 19968+18	.17182+18	.14811+18	.12791+18	. 11066+18	.95894+17	.83242+17	. 72378+17	.63032+17	.54980+17	.48030+17
HEIGHT (METERS)	30000.	31000.	32000.	.33000.	34000.	35000.	36000.	37000.	38000.	39000.	40000.	41000.	42000.	43000.	44000.

TABLE 5. • (contd)

INTEGRATED NUMBER DENSITY VS. HEIGHT

HEIGHT	NUMBER	ZENITH INT NO DEN	HORIZONTAL INT NO DEN	RELATIVE HORIZONTAL	HORIZONTAL INT NO DEN
(METERS)		CNO/SG-CMO	CNO/SG-CM)	INT NO DEM	DERIVATIVE
45000.	. 42021+17	.34480+23	.24089+25	.11185+00	14037-04
46000.	.36818+17	.30543+23	.21270+25	.98760-01	12189-04
47000.	. 32306+17	.27093+23	.18824+25	.87405-01	10518-04
48000.	.28572+17	.24056+23	.16718+25	.77626-01	91544-05
49000.	.25365+17	.21363+23	.14860+25	.68997-01	81166-05
50000.	.22519+17	.18971+23	.13212+25	.61347-01	71972-05
51000.	.19993+17	.16848+23	.11753+25	.54570-01	63722-05
52000.	.17751+17	.14963+23	.10462+25	.48575-01	56315-05
53000.	.15762+17	.13289+23	.93237+24	.43292-01	49169-05
54000.	.14114+17	.11800+23	.83349+24	.38701-01	43316-05
55000.	.12708+17	.10460+23	.74468+24	.34577-01	39219-05
56000.	.11423+17	.92545+22	.66415+24	.30838-01	35574-05
57000.	.10251+17	.81717+22	:59124+24	.27452-01	32184-05
58000.	.91827+16	.72008+22	.52532+24	.24392-01	29071-05
59000.	.82106+16	.63319+22	.46583+24	.21630-01	26214-05

TABLE 5. (contd)

INTEGRATED NUMBER DENSITY VS. HEIGHT **

ER ZENITH HORIZONTAL RELATI

HORIZONTAL INT NO DEN DERIVATIVE	23595-05	21200-05	19013-05	17018-05	15201-05	13551-05	12053-05	10697-05	94711-06	83656-06	73708-06	64773-06	56765-16	45603-06	43223-06
RELATIVE HORIZONTAL INT NO DEN	19141-01	.16903-01	.14894-01	.13094-01	.11485-01	.10049-01	.87695-02	.76332-02	.66258-02	.57350-02	.49490-02	:42575-02	.36505-0	.31193-02	.20558-02
HORIZONTAL INT NO DEN CNO/SO-CMO	.41224+24	.36404+24	.32077+24	.28201+24	.24734+24	.21641+24	.18887+24	.16439+2+	4270+24	.12351+24	.10659+24	91.92+23	.786:9423	.67130+23	5:198+23
ZENITH INT NO DEN CNO/SO-CM)	.55557+22	.48637+22	. 42479+22	.37011+22	.32166+22	.27882+22	.24103+22	.20778+22	.17867+22	.15305+22	.13073+22	11131-72	.94442+21	.79844+2:	.7250 28
NUMBER DENSITY CNO/CU-CMO.	.73275#16	.65266+:6	.50014+16	.51460+16	.45546+16	.40221+16	91+00th00.	0 1+10 + 10 mg.	.27301+16	.23870+16	20813+16	.18095+16	.15,85+16	. 13354+16	.11674+16
HEIGHT (METERS)	£0000.	61000.	.00009	.00000	64000.	.5000.	66000.	.0000	.90089	.00069	70000.	71000.	. 7:000.	7300	74000.

TABLE 5. (contd)

INTEGRATED NUMBER DENSITY US. HENCHT

HOR ZONTA: INT NO DEN - 37 54-0 DERIVATIVE -. 32528-06 -.28095-06 -. 2420 3-06 -. 20888-06 -. 18109-06 -.:5211-06 -.12355-06 -. 10107-06 -.82693-07 -.67651-07 -. 553e9-07 -. 45311-07 -. 37055-07 -. 304 : 8-E7 .19026-02 .50303-03 INT NO EN .22525-02 .15999-02 .13388-02 .11140-02 .91951-03 .75188-03 .61497-03 .41145-03 .33652-03 27523-03 .22504-03 .18398-03 .15033-03 HORIZONTAL RELATIVE .72476422 HORIZONTEL INT NO PEN .19803+23 ,10834+23 . 48511+23 .40976+23 34458+23 .28834+23 .23992+23 .16193+23 .13244+23 88613+22 .48:66+22 CNO. 50-CM3 39624+22 . 32377+22 59275+22 ZENITH INT NO DEN .98377+20 .80499+20 .65875+20 .53914+20 .361 8+10 CNO-SQ-CM> 44127+20 .56420+21 471-11-21 .392i7+21 .32479+21 .26771+21 21956+21 .17961+21 .14695+21 .12023+21 .10031+16 .85710+15 73040+15 , 61990+15 .52400+15 44100+15 36060+15 .29490+15 ,24120+15 . 19730+15 . 16140+15 13210+15 .10800+15 . 72330+14 CNO-UD/OND .88400+14 NU BER DENSITA 80000° 60009 CMETERSO 75000. 77000. 78000. 79000. 21000. 82000. 83000. 84000. 86000. 88000. 35000. 87000. 89000. HEIGHT

TABLE 5. (contd)

INTEGRATED NUMBER DENSITY US. HEIGHT

HORIZONTAL INT NO DEN	DEKINHII E	יח-ונניסי	10-12102.	10752-01	70-0400-0	- 0/51-07	80-086	30-39/49-	56604-08	×0-91194.1	30936-08	
PELGTIVE HORIZONTAL INT MG DEN	.12273-0	\$3-986°.	.31093-04	65991-04	25.00 - 40 - 60 - 60 - 60 - 60 - 60 - 60 -	44297-0	26 0 0 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2	40-07000:	25109-04	.20929-34	.1750 -04	
HORIZONTAL INT NO DEN CNO/SQ-CMO	26431+22	.21534+22	. 17465+22	.14212+22	.11619+22	.95393+21	.78673-21	.6-091+21	.54077+2	.45074+21	.37692+21	
ZENITH INT N° DEN	.29565+20	.24203+20	.19837+20	.16330*10	.13502+20	. 1.210+20	.93447+19	.7319" +19	. 5665+19	.55330+19	.46781+19	
NUMBER DENSITY CNO/CU-CM)	.59180+14	.48430+14	. 39040+14	.31410+14	.25400+14	.20630+14	.16840+14	.13800+14	1:36(+:4	.97840+13	.77830+13	
H"IGHT CMETERSO	90000.	91000.	92000.	÷3000°	94000.	020	96000.	97000.	98000	00 65	100000.	

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ACKNOWLEDGMENT

The authors wish to acknowledge the help of Robert Dancey in programming the computer for the computations of the tables.

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region of the spectrum is attenuated principally by Rayleigh scattering. The attenuation is a function of the total number of molecules, or, if the molecular weight remains constant, of the air mass along the path of the ray that is tangential to any level, ho, above the surface.

Tables are given in terms of mass of air and number of molecules for a vertical path originating at h_o , a horizontal path tangent to h_o , the horizontal quantities relative to a unit atmosphere, and the rate of change with respect to h_o . The tables cover

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Tables are given in terms of mass of air and number of molecules for a vertical path originating at h, a horizontal path tangent to h, the horizontal quantities relative to a unit atmosphere, and the rate of change with respect to h. The tables cover

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U. S. Naval Ordnance Test Station Horizontal Air-Mass Tables (Card 2) an altitude range of 0 to 100 km, calculated for an atmosphere terminated at 200 km. These tables are intended primarily for use above a nominal altitude of 20 km, where refraction, which was not allowed the community of the part of the community of the contraction of the contra	
	Horizontal Air-Mass Tables an Eltitude range of 0 to 100 km, atmosphere terminated at 200 km. intended primarily for use above of 20 km, where refraction, which of the terminations has a new conventations.

Atmospheric optics; air-mass tables; satellite illumination; satellite photometry; Rayleigh scattering; Echo satellite

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an altitude range of 0 to 100 km, calculated for an atmosphere terminated at 200 km. These tables are intended primarily for use above a nominal altitude of 20 km, where refraction, which was not allowed for in the computations, has a negligible effect.

Atmospheric optics; air-mass tables; satellite illumination; satellite photometry; Rayleigh scattering; Echo satellite

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an altitude range of 0 to 100 km, calculated for an atmosphere terminated at 200 km. These tables are intended primarily for use above a nominal altitude of 20 km, where refraction, which was not allowed for in the computations, has a negligible effect.

Atmospheric optics; air-mass tables; satellite illumination; satellite photometry; Rayleigh scattering; Echo satellite

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